

Quick Summary:

$$\begin{aligned} \text{Work} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (\text{FORCE})(\text{DIST}) \\ &= \int_a^b (\text{FORCE})(\text{DIST}) \end{aligned}$$

Problem type 1: (Leaky bucket/spring)

$$\text{FORCE} = f(x_i), \quad \text{DISTANCE} = \Delta x,$$

Problem type 2: (Chain/pumping)

FORCE = weight of each horizontal slice

DISTANCE = distance moved by a slice

For a chain, we have

k = density = force per distance

FORCE = weight of slice = $k\Delta x$

DISTANCE = distance moved by slice

For pumping, we have

k = weight per volume

FORCE = $k(\text{area of horiz. slice})\Delta y$

DISTANCE = distance moved by slice

Some unit facts:

	Metric	Standard
Mass	kg	
Accel.	9.8 m/s ²	32 ft/s ²
Force	Newtons N = kg·m/s ²	pounds = lbs
Dist.	m = meters	ft = feet
Work	Joules J = N·m	foot-pounds ft·lbs

g = grams, in = inches, yd = yards, mi = miles

1000 g = 1 kg

100 cm = 1 meter

12 inches = 1 foot

3 feet = 1 yard

5280 ft = 1 mile

Density of water = 1000 kg/m³ = 9800 N/m³
= 62.5 lbs/ft³

Review: Particular scenarios

Type 1 Problems:

$$\text{FORCE} = f(x_i), \text{DISTANCE} = \Delta x$$

1. HW 4A/1, 2, 8, 9 and HW 4C/1

Given force, just need to integrate!

$$\text{Work} = \int_a^b f(x) dx$$

2. HW 4A/3, 4 (Springs)

(i) Covert all to meters

(ii) Label natural length, L , and note that L corresponds to $x = 0$.

$$\text{Force} = f(x) = kx$$

$$\text{Work} = \int_a^b kx dx$$

Step 1: Find k

Step 2: Answer question.

Type 2 Problems:

FORCE = weight of a horizontal slice,

DISTANCE = distance to top

3. HW 4A/5 and HW 4C/2 (Chain)

(i) k = density of chain = weight/dist

(ii) FORCE at a subdivision = $k\Delta x$

(iii) Label top $x=0$, then DIST = x_i .

$$\text{Work} = \int_a^b x k dx$$

4. HW 4A/6,7 and HW 4C/3 (Pumping)

Water density = $9800 \text{ N/m}^3 = 62.5 \text{ lbs/ft}^3$

(i) Label (put in xy -plane)

(ii) Draw a horizontal slice and find a formula for its area.

(iii) FORCE = (Density)(Area) Δy

(iv) DIST = distance to top

$$\text{Work} = \int_a^b (\text{Dist})(\text{Density})(\text{Area}) dy$$